12.003 Physics of Atmospheres and Oceans

Problem Set 3: Convection and moist processes

Due date: Friday 6th October, 2006

1. Make sure that you have read and studied Chapters 1 to 4 of our notes.

2. In class, we showed that the pressure of an isothermal atmosphere varies exponentially with height. Consider now an atmosphere with uniform potential temperature. Find how pressure varies with height, and show in particular that such an atmosphere has a discrete top (where $p \to 0$) at altitude $RT_o/(\kappa g)$, where $R$, $\kappa$, and $g$ have their usual meanings, and $T_o$ is the temperature at 1000hPa pressure.

3. Consider a horizontally uniform atmosphere. The surface temperature is 20°C; immediately above the surface, the air temperature falls to 10°C, and falls off thereafter with height at a rate of $5Kkm^{-1}$.

   (a) Suppose the surface air is perfectly dry (relative humidity 0%). Show that this temperature profile is convectively unstable and determine the altitude to which surface air will rise (assuming that the air parcels will stop rising when they reach their level of zero buoyancy).

   You may assume that unsaturated air parcels follow the dry adiabatic lapse rate of $10Kkm^{-1}$ under all conditions.

   (b) Suppose now that the surface air has a relative humidity of 50% (i.e., its specific humidity is 50% of the saturated value). From the table of $q_s(T)$ below, use the saturation specific humidity $q_s$ at the surface temperature (20°C) and hence determine the specific humidity of the surface air. Also from the table of $q_s(T)$, estimate the temperature at which this air will saturate, and hence determine the condensation level. Determine to what altitude convection will occur for this case.

   You may assume that saturated air parcels follow a moist adiabat with a lapse rate of $7Kkm^{-1}$.

   The saturation specific humidity $q_s$ depends on temperature as given in the following table (interpolate as necessary).

<table>
<thead>
<tr>
<th>$T$(°C)</th>
<th>$q_s$(g kg$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3.8</td>
</tr>
<tr>
<td>4</td>
<td>5.1</td>
</tr>
<tr>
<td>8</td>
<td>6.7</td>
</tr>
<tr>
<td>12</td>
<td>8.7</td>
</tr>
<tr>
<td>16</td>
<td>11.3</td>
</tr>
<tr>
<td>20</td>
<td>14.5</td>
</tr>
</tbody>
</table>
4. Consider an atmospheric temperature profile at dawn with a temperature discontinuity (inversion) at 1km, and a tropopause at 11km, such that

\[
T(z) = \begin{cases} 
10^\circ\text{C}, & z < 1\text{km} \\
[15 - 8(z - 1)]^\circ\text{C} & 1 < z < 11\text{km} \\
-65^\circ\text{C} & z > 11\text{km}
\end{cases}
\]

(where here \(z\) is expressed in km). Following sunrise at 6 a.m. until 1 p.m., the surface temperature steadily increases from its initial value of 10\(^\circ\)C at a rate of 3\(^\circ\)C per hour. Assuming that convection penetrates to the level at which air parcels originating at the surface attain neutral buoyancy, describe the evolution during this time of convection

i) if the surface air is completely dry;

ii) if the surface air is saturated.

You may assume that unsaturated (saturated) air parcels follow the dry (moist) adiabatic lapse rate of 10 K km\(^{-1}\) (7 K km\(^{-1}\)) under all conditions (even at finite displacements).

5. Consider an isothermal atmosphere in hydrostatic balance, with temperature \(T_0 = 280\text{ K}\). Suppose an air parcel is moved adiabatically from the surface to an altitude of 10 km. Determine its temperature on arrival

i) assuming its potential temperature is conserved during adiabatic displacement;

ii) assuming its temperature decreases with altitude at the adiabatic lapse rate \(\frac{\partial T}{\partial z} = -g/c_p\).

Compare and discuss. Now repeat the calculations for an atmosphere which has the same surface temperature, but in which the potential temperature is constant with height (you should be able to make use of your answer to question 2 here).

6. Consider the convective circulation shown below.

Air rises in the center of the system; condensation occurs at altitude \(z_B = 1\text{km}\) \((p_B = 880\text{hPa})\), and the convective cell (cloud is shown by the shading) extends up to \(z_T = 9\text{km}\) \((p_T = 330\text{hPa})\), at which point the air diverges and descends adiabatically in the downdraft region. The temperature at the condensation level, \(T_B\), is 20\(^\circ\)C. Assume all condensate falls out immediately as rain.
(a) Determine the specific humidity at an altitude of 3 km within the cloud.

(b) The upward flux of air, per unit horizontal area, through the cloud at any level 
\(z\) is \(w(z)\rho(z)\), where \(\rho\) is the density of dry air and \(w\) the vertical velocity. Mass 
balance requires that this flux be independent of height within the cloud. Con-
sider the net upward flux of water vapor within the cloud and hence show that 
the rainfall rate below the cloud (in units of mass per unit area per unit time) 
is \(w_B\rho_B[q_{sB} − q_{sT}]\), where the subscripts “\(B\)” and “\(T\)” denote the values at cloud 
base and cloud top, respectively. If \(w_B = 5 \text{ cm s}^{-1}\), and \(\rho_B = 1.0 \text{ kg m}^{-3}\), deter-
mine the rainfall rate in \(\text{ cm per day}\).

7. Use the expression for saturated specific humidity:

\[
q_s = \left( \frac{R}{R_v} \right) \frac{e_s(T)}{p}
\]

(Eq.(4.23) in our notes) and the following empirical (but pretty accurate) relation for satu-
rated vapor pressure \(e_s(T)\):

\[
e_s = A \exp (\beta T)
\]

where \(A = 6.11 \text{ mb}\) and \(\beta = 0.067 \text{ °C}^{-1}\) and \(T\) is in \(\text{ °C}\), to compute from tabulated data 
of \(T(p)\) (use annual means), vertical profiles of saturated specific humidity, \(q_s(p)\). Carry 
out your calculation using \(T(p)\) data at the equator and at 50°N. You will need to look up 
values of \(R\) and \(R_v\) in your notes.

Compare your \(q_s\) profiles with observed profiles of \(q\), the specific humidity, at the same 
latitudes. Comment?

You can obtain tabulated data here (follow detailed instructions in appendix):
http://paoc.mit.edu/labweb/atmos-obs/temperature.htm
and
http://paoc.mit.edu/labweb/atmos-obs/specifichumidity.htm

**Appendix**

To obtain data from the web, follow this example.

Suppose you want to obtain a table of annual mean \(T(p)\) at 50°N.

Go to http://paoc.mit.edu/labweb/atmos-obs/observations.htm and click on ‘temperature’
on the side bar. Click on the annual mean T picture and you will taken through to ‘Ingrid’,
a data server at Lamont.

Underneath the figure, in the ‘Get Data’ row, click ‘Entire data set’, then click ‘tables’
toward the top on the right) and then P,Y Table.

The tabulated data should pop up in your browser. Pick out the 50°N (not south!!) row. 
That’s it.

The procedure is the same for other data.
E-mail me jmarsh@mit.edu if you have problems.