There are five questions in all. Answer any three questions. Only three questions will be graded—do not leave any ambiguity about which questions you have answered.

In your answers, be sure to state any assumptions you make.

You should have 7 numbered pages, including one page giving values of constants and other information you may need.
1 Define planetary emission temperature and describe what it represents.

(a) Suppose that the Earth’s rotation axis were normal to the Earth-Sun line. The solar flux, measured per unit area in a plane normal to the Earth-Sun line, is $S_0$. By considering the solar flux incident on a latitude belt bounded by latitudes $(\varphi, \varphi + d\varphi)$, show that $F$, the 24hr-averaged solar flux per unit area of the Earth’s surface, varies with latitude as

$$F = \frac{S_0}{\pi} \cos \varphi .$$

(b) Using the result given in part (a), suppose that the atmosphere is completely transparent to solar radiation, but opaque to infrared such that, separately at each latitude, the radiation budget can be represented as for a “single slab” atmosphere (as shown in the figure below). Determine how surface temperature varies with latitude.

(c) Calculate the surface temperature at the equator, $30^\circ$, and $60^\circ$ latitude, if Earth albedo is $30\%$ and $S_0 = 1367W\ m^{-2}$. 

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[Diagram showing radiation budget with labels for solar input, reflected shortwave, radiated up from ground, radiated down to ground, and space radiation.]
2. (a) Write down the equation of hydrostatic balance and explain its meaning.

(b) Show that the “thickness” of a layer of atmosphere, sandwiched between two pressure surfaces, \( p_1 \) and \( p_2 \) is given by:

\[
z_2 - z_1 = R \int_{p_2}^{p_1} \frac{T \, dp}{g \, p}.
\]

where \( R \) is the gas constant and \( g \) the acceleration due to gravity. Use this expression to:

i. deduce how pressure varies with height in an isothermal atmosphere. If the temperature of such an atmosphere is \( 0^\circ \text{C} \), at what altitude does pressure equal 100mb (assuming surface pressure is 1000mb)?

ii. deduce that atmospheric layers are ‘thick’ in tropical regions and ‘thin’ in polar regions.

iii. estimate how much the 100mb surfaces slopes down from equator to pole, given that the tropical troposphere is typically \( 40^\circ \text{C} \) warmer than the poles.
3 Radial inflow experiment

The figure below shows the trajectory of a fluid parcel (in the rotating frame) obtained from the radial inflow experiment discussed in class.

If parcels of fluid are endowed by the axial angular momentum of the rotating frame as they enter the tank at the outer radius \( r_1 \) and subsequently conserve angular momentum as they flow inwards:

(a) show that they acquire a sense of rotation which is the same as that of the rotating table but which is greatly magnified at small radius \( r \).

(b) show that the Rossby number is given by

\[
R_o = \frac{v_\theta}{2\Omega r} = \frac{1}{2} \left( \frac{r^2}{r_1^2} - 1 \right)
\]

where \( r_1 \) is the outer radius of the tank, \( v_\theta \) is the azimuthal speed of the current and \( \Omega \) is the rotation rate of the apparatus.

(c) sketch a graph of the Rossby number as a function of \( \frac{r}{r_1} \), indicating regions where the flow is in geostrophic and cyclostrophic balance.

(d) discuss ways in which the radial inflow experiment is a useful analogue of balanced motion in the atmosphere and ocean.

1.4
4 The figure below shows the observed net radiation at the top of the atmosphere, as a function of latitude. Taking this as a starting point, describe the chain of dynamical processes that leads to the existence of anticyclonic circulation gyres in the upper subtropical oceans. Be sure to discuss the key physical mechanisms and constraints involved in each step. You do not need to include any mathematics in your discussion, but feel free to do so if it will help your discussion.
5 Write notes on three of the following topics:

(a) the moist adiabatic lapse rate
(b) the Taylor-Proudman theorem
(c) the Coriolis force
(d) differentiation following the motion
(e) Ekman layers in the ocean
(f) the zonal-average temperature and salinity distribution within the ocean
INFORMATION YOU MAY NEED

Constants

Coefficient of thermal expansion of water $\alpha_w = 2.0 \times 10^{-4} \text{K}^{-1}$.  
Density of dry air at STP $\rho_s = 1.29 \text{ kg m}^{-3}$
Density of water at 0°C $\rho_0 = 1000 \text{ kg m}^{-3}$
Earth mean radius $a = 6370 \text{ km}$
Earth rotation rate $\Omega = 7.27 \times 10^{-5} \text{s}^{-1}$
Gas constant for dry air $R = 287 \text{ JK}^{-1}\text{kg}^{-1}$
Gravitational acceleration $g = 9.81 \text{ m s}^{-2}$
Planetary albedo of the Earth $\alpha_p = 0.30$
Solar flux at radius of Earth’s orbit $S_0 = 1367 \text{ Wm}^{-2}$
Specific heat of air at constant pressure $c_p = 1005 \text{ J kg}^{-1}\text{K}^{-1}$
Specific heat of water $c = 4187 \text{ J kg}^{-1}\text{K}^{-1}$
Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$
$1^\circ$ latitude $\approx 111 \text{ km}$
Beta effect $\beta = 10^{-11} \text{s}^{-1}\text{m}^{-1}$
Coriolis parameter $f = 10^{-4} \text{s}^{-1}$
Depth of ocean $H = 4 \text{ km}$