Interpretation of the propagation of surface altimetric observations in terms of planetary waves and geostrophic turbulence

Ross Tulloch

Center for Atmosphere Ocean Science, Courant Institute of Mathematical Sciences, New York University

John Marshall

Department of Earth, Atmospheric and Planetary Sciences, Massachusetts Institute of Technology

K. Shafer Smith

Center for Atmosphere Ocean Science, Courant Institute of Mathematical Sciences, New York University

R. Tulloch, Center for Atmosphere Ocean Science, Courant Institute, New York University, 251 Mercer St., New York, NY 10012, USA. (tulloch@cims.nyu.edu)

J. Marshall, Department of Earth, Atmospheric and Planetary Sciences, Massachusetts Institute of Technology, 77 Massachusetts Ave., Cambridge, MA 02139, USA. (jmarsh@mit.edu)

K. S. Smith, Center for Atmosphere Ocean Science, Courant Institute, New York University, 251 Mercer St., New York, NY 10012, USA. (shafer@cims.nyu.edu)
Abstract. The interpretation of surface altimetric signals in terms of Rossby waves is revisited. Rather than make the long-wave approximation, the horizontal scale of the waves is adjusted to optimally fit the phase speed predicted by linear theory to that observed by altimetry, assuming a first baroclinic mode vertical structure. It is found that in the tropical band, the observations can be fit if the wavelength of the waves is assumed to be large, of order 600 km or so. However polewards of ±30° it is difficult to fit linear theory to the observations: the required scale of the waves must be reduced to about 100 km, somewhat larger than the local deformation wavelength. It is argued that these results can be interpreted in terms of Rossby wave, baroclinic instability and turbulence theory. At low latitudes there is an overlap between geostrophic turbulence and Rossby wave times scales and so an upscale energy transfer from baroclinic instability at the deformation scale produces waves. At high latitudes there is no such overlap and waves are not produced by upscale energy transfer. These ideas are tested by using surface drifter data to infer turbulent velocities and timescales which are compared to those of linear Rossby waves. A transition from a field dominated by waves to one dominated by turbulence occurs at about ±30°, broadly consistent with the transition that is required to fit linear theory to altimetric observations.
1. Introduction

Altimetric observations of sea surface height (SSH) of the ocean show westward propagating phase anomalies in all of the major oceans except the Antarctic Circumpolar Current (ACC), the Kuroshio and the Gulf Stream, where the propagation is eastward. Chelton and Schlax [1996] attempted to understand these observations in terms of linear, first baroclinic Rossby waves in a resting ocean and in the long-wave limit. They found that observed phase speeds were larger than predicted by theory outside the tropics by as much as a factor of two [see, for example, the introduction of Colin de Verdière and Tailleux, 2005, who review an extensive literature on the subject]. Chelton et al. [2007] recently observed that SSH variability appears to be nondispersive and consistent with the behavior of nonlinear eddies in many regions of the world ocean, particularly poleward of 25°, in western basins and in the ACC. Some of the discrepancy between the observations and linear theory can be resolved by including mean flow and topography [Killworth et al., 1997; Dewar and Morris, 2000; Killworth and Blundell, 2005; Maharaj et al., 2007]. We find that the “fit” of linear theory to observations at high latitudes is not as successful as at low latitudes. However, the downstream phase speed observed in the ACC is captured rather well. Less satisfying, is the mismatch of best fit speeds to observations in the 40° to 50° latitude bands. Killworth and Blundell [2005] appear to obtain a closer fit there, the reasons for which are not clear to us.

A number of authors adopt the planetary geostrophic approximation (Dewar, 1998, appendix; Killworth and Blundell, 1999, 2003; Colin de Verdière and Tailleux, 2005) and so automatically make the long-wave approximation by neglecting relative vorticity. Others have considered Rossby basin modes in the quasigeostrophic approximation [Cessi and Primeau, 2001; LaCasce
and Pedlosky, 2004]. As noted by Killworth and Blundell [2005], all such calculations implicitly assume production of waves at the eastern boundary, yet their ray tracing calculations through the observed hydrography indicate that such waves are generally unable to cross the basin. Instead, Killworth and Blundell [2007] investigate the assumption that waves are produced throughout the ocean via local forcing by winds, buoyancy exchange or baroclinic instability of the mean state; they put this assumption to use by computing the dispersion relation at each lateral position, assuming local forcing and horizontal homogeneity (i.e. doubly-periodic boundary conditions for each $1° \times 1°$ section, the ‘local approximation’).

Both Killworth and Blundell [2007] and [Smith, 2007] (in a similar analysis) find that the oceans are rife with baroclinic instability, occurring at or below the deformation scale, thus providing a ready source of energy, cascading upscale from below, for the waves and turbulence seen at the ocean’s surface. Indeed both altimeter observations and numerical ocean models provide evidence of an inverse spectral flux of kinetic energy from the deformation scale up to an arrest wavelength of order 500–1000 km, which decreases with latitude but does not scale closely with the deformation scale [Scott and Wang, 2005; Schlösser and Eden, 2007]. Such an inverse cascade\(^1\) is possibly the result of nonlinear interactions in geostrophic turbulence. The inverse cascade can be arrested or slowed before reaching the basin scale by Rossby waves [Rhines, 1975], stratification $N^2(z)$ [when energy is contained in baroclinic modes, particularly if $N^2$ is surface intensified as in the ocean — see Fu and Flierl, 1980; Smith and Vallis, 2001] or dissipative processes [Arbic and Flierl, 2004; Thompson and Young, 2006]. It is not yet clear which of these processes, if any of them, sets the ultimate arrest scale.

Rhines [1975] theorized that, because the eddy timescale increases as the spatial scale grows in the inverse cascade, a transition will occur at the spatial scale where the eddy timescale
matches that of Rossby waves with the same spatial scale. The transition scale, commonly referred to as the Rhines scale, is $L_R \sim (2u_t/\beta)^{1/2}$, where $u_t$ is the square root of the eddy kinetic energy (which, in the two-dimensional system considered, is the only energy). It is at this spatial scale, Rhines suggested, that the turbulent energy is shunted into either jets or waves, or both, depending on the strength and homogeneity of the eddy field. Numerical experiments presented in Rhines’ paper demonstrate that, even when the eddies are energetic enough to form jets, Rossby waves may also be energized. Vallis and Maltrud [1993] refined the idea of a wave-turbulence crossover by noting that while the Rhines effect cannot halt the cascade alone, it inhibits energy transfer into a dumbbell-shaped region around the origin in wavenumber space, which leads to the generation of zonally elongated flow. There is some evidence for zonal jet formation in the ocean [Maximenko et al., 2005; Richards et al., 2006], perhaps a signature of the Rhines effect, in addition to the observations of waves by Chelton and Schlax [1996] and Chelton et al. [2007].

Recent research [Theiss, 2004; Smith, 2004] has suggested that, on the giant gas planets, turbulent generation at small scales should result in jet formation in regions equatorward of some critical latitude, and a more isotropic eddy field in regions poleward of that critical latitude. Scott and Polvani [2007] confirmed that a critical latitude for jet formation does arise in direct numerical simulations of forced-dissipative shallow-water turbulence on the sphere. Theiss [2006] extends the idea further by replacing $\beta$ with the mean flow-dependent meridional potential vorticity (PV). Specifically, he derives a “generalized” Rhines scale, which includes the effect of mean shears, and a corresponding critical latitude, polewards of which jets do not form.

Following on these ideas, Eden [2007] analyzed eddy length scales in the North Atlantic Ocean both via satellite altimetry and an eddy resolving primitive equation model. At high latitudes,
he shows evidence that eddy scales vary with the Rossby deformation radius, consistent with Stammer [1997], while at low latitudes eddy scales are consistent with a generalized Rhines scale. That is, eddies scale with the smaller of the deformation radius and the Rhines scale, with a critical latitude near 30°N, where the deformation scale is similar to the Rhines scale.

In this paper we reinterpret sea surface height (SSH) signals in the context of the aforementioned theoretical ideas. Specifically, we avoid the issue of jet formation, but posit that below a critical latitude baroclinic eddies transform some of their energy into Rossby waves, and that these waves dominate the surface height field. At higher latitudes, where Rossby wave frequencies are too small to be excited by the inverse cascade, the surface height field remains turbulent.

We investigate this hypothesis as follows. Following Killworth and Blundell [2007], we compute the local Rossby wave dispersion, but rather than make the long-wave approximation, we adjust the horizontal scale of first baroclinic waves to best-fit the observed phase speeds, and thereby infer a length scale for the waves. In the tropics the fitted wavelength is close to both the Rhines scale and previously observed SSH scales. Outside the tropics, it is either impossible to match the observed phase speeds with Rossby wave speeds at any wavelength (probably because linear theory is inadequate) or the fitted wavelength lies near the deformation scale. Using surface drifter data to estimate the eddy timescale and energy level, we show that at high latitudes the turbulent timescale is faster than the Rossby wave timescale, so turbulence dominates, but at low latitudes the Rossby wave and turbulent timescales overlap, enabling the excitation of waves by turbulence.

In section 2 we compare first baroclinic Rossby wave phase speeds calculated with the Forget [2008] ocean atlas (essentially, a mapping of Argo and satellite altimetric data using interpolation by the MITgcm) with observed altimetric phase speeds provided by Chris Hughes. We repeat the
calculations of Chelton and Schlax [1996] and Killworth and Blundell [2005] with mean flow and stratification, and topographic slopes. We arrive at a conclusion consistent with Chelton et al. [2007] – in mid-latitudes, phase speeds predicted by long-wave linear theory are typically faster than observed phase speeds. In section 3, where possible, we fit the phase speeds predicted by the linear model to observed phase speeds by adjusting the horizontal scale of the waves. We obtain a marked meridional variation in the scale of the fitted waves: equatorwards of ±30° the fitted scale is large and gradually decreases with latitude, having an implied Rhines wavelength of about 600 km. Polewards of ±30° the linear fit begins to fail, and eventually fitted scales match the deformation scale. In section 4 we interpret our result via a comparison of turbulent and wave timescales. Finally we estimate the critical latitude at which waves give way to turbulence by making use of surface eddy velocities from drifter data, provided by Nikolai Maximenko. In section 5 we conclude.

2. Linear Rossby waves

Rossby waves result from the material conservation of potential vorticity (PV) in the presence of a mean gradient. As a parcel moves up or down the background mean PV gradient, its own PV must compensate, generating a restoring force toward the initial position. The result is a slow, large-scale westward propagating undulation of mean PV contours. Mean currents change the structure of the waves in two ways: by altering the background PV gradient (sometimes so much so that β is negligible), and by Doppler shifting the signal. A number of authors [Killworth et al., 1997; Dewar and Morris, 2000; Killworth and Blundell, 2005; Maharaj et al., 2007] have shown that the straightforward inclusion of the mean thermal-wind currents in the linear Rossby wave problem leads to a much closer agreement between the observed phase speeds and theory. Here we take an approach closest to Killworth and Blundell [2007] [see also Smith, 2007] and
compute phase speeds in the local quasigeostrophic approximation, using the full background shear and stratification in a global hydrographic dataset. Our focus, however, is on attempting to fit the linear results to the satellite data and thereby determining the limitations of linear wave theory when mean effects are fully included, and characterizing the scale of the waves that are consistent with the observed phase speeds. We now briefly outline the approach, relegating details to the appendix.

We assume, away from coasts, that the Rossby wave and eddy dynamics of the ocean at each latitude, longitude coordinate may be represented by the linearized inviscid quasigeostrophic equations on a $\beta$-plane with slowly varying local mean velocity [see Pedlosky, 1984]. In the interior, linear QG potential vorticity is linearly advected by the mean flow

$$\partial_t q + \mathbf{U} \cdot \nabla q + \mathbf{u} \cdot \nabla Q = 0, \quad -H < z < 0,$$

where $\mathbf{U} = U(x, y, z) \hat{x} + V(x, y, z) \hat{y}$ is the local mean velocity, $q = \nabla^2 \psi + (f^2/N^2 \psi_z) z$ is the quasigeostrophic potential vorticity (QGPV), $f$ is the local Coriolis parameter, $H$ is the local depth of the ocean, $N^2(z) = -(g/\rho_0) d\rho/dz$, and the eddy velocity is $\mathbf{u} = -\psi_y \hat{x} + \psi_x \hat{y}$. The mean QGPV gradient $\nabla Q$, includes horizontal shear$^2$, and is given by

$$\nabla Q = \left[ V_{xx} - U_{yx} + \left( \frac{f^2}{N^2} V_z \right)_z \right] \hat{x} + \left[ \beta + V_{xy} - U_{yy} - \left( \frac{f^2}{N^2} U_z \right)_z \right] \hat{y}.$$  

At the rigid lid upper boundary buoyancy is linearly advected

$$\partial_t b + \mathbf{U} \cdot \nabla b + \mathbf{u} \cdot \nabla B = 0, \quad z = 0,$$

where the buoyancy anomaly is defined as $b = f\psi_z = -g\rho/\rho_0$, the mean buoyancy is $B = -g\bar{\rho}/\rho_0$, so the mean buoyancy gradient, via thermal wind balance, is $\nabla B = fV_z \hat{x} - fU_z \hat{y}$.

Slowly varying bottom topography is included using the approach of Smith [2007], using the Smith and Sandwell [1997] global seafloor topography dataset, see the appendix for details.
Assuming a wave solution \( \psi(x, y, z, t) = \Re\{\hat{\psi}(z) \exp [i(kx + \ell y - \omega t)]\} \), and likewise for \( q \) and \( b \), one obtains the linear eigenvalue problem,

\[
\begin{align*}
(K \cdot U - \omega_n) \hat{b}_n &= (\ell B_x - k B_y) \hat{\psi}_n, \quad z = 0, \\
(K \cdot U - \omega_n) \hat{q}_n &= (\ell Q_x - k Q_y) \hat{\psi}_n, \quad -H < z < 0,
\end{align*}
\]  

(4a)

(4b)

where \( K = (k, \ell) \), and \( \hat{\psi}_n \) is the \( n \)th eigenvector, sometimes called a ‘vertical shear mode’, and \( \hat{q}_n \) and \( \hat{b}_n \) are linear functions of \( \hat{\psi}_n \). (The hat notation implies dependence on the wavenumber \( K \), which is suppressed for clarity). The eigenvalues \( \omega_n \) are the frequencies of the wave solutions, with the real part resulting in phase propagation and imaginary parts, if they exist, producing growth or decay of the wave. The problem is discretized in the vertical using a layered formulation; in the discretized case, there are as many shear modes as there are layers. The expressions for the discrete surface buoyancy \( \hat{b} \) and \( \hat{q} \) in terms of \( \hat{\psi} \), and other details of the discretization can be found in the appendix, and in Smith [2007].

Equation (4) is solved by first considering the neutral modes, which diagonalize the vertical derivatives in the stratification operator as follows. For a resting ocean (\( U = 0 \), implying \( B_x = B_y = Q_x = 0 \) and \( Q_y = \beta \)), equation (4) reduces to the standard Rossby wave dispersion relation

\[
\omega_m = -\frac{k \beta}{K^2 + K_m^2},
\]  

(5)

where \( K = |K| \) and \( K_m \) is the \( m \)th deformation wavenumber, which is given by the following Sturm Liouville problem

\[
\frac{d}{dz} \left( \frac{f^2}{N^2} \frac{d\Phi_m}{dz} \right) = -K_m^2 \Phi_m, \quad \frac{d\Phi_m}{dz} \bigg|_{z=0} = \frac{d\Phi_m}{dz} \bigg|_{z=-H} = 0.
\]  

(6)

The eigenfunctions \( \Phi_m \) are often called the ‘neutral modes’; they form an orthonormal basis of the vertical structure in a resting ocean.
The mean velocity and buoyancy fields are computed from the ocean atlas of Forget [2008], as described in the Appendix, and these are used to construct the mean buoyancy and PV gradients. At each lateral position in the ocean, we then compute the neutral modes and their deformation scales from (6), as well as $\omega_n$ and $\hat{\psi}_n$ from the complete dispersion relation (4). We denote the zonal phase speed of this mode as

$$c_R = \frac{\omega_n}{k}.$$

### 2.1. Observations of phase propagation from altimetry

In the long-wave, resting ocean limit, the dominant first baroclinic mode has a westward phase speed given by equation (5) with $K = 0$, so $c_R = \beta/K^2$. A zonal average of the long-wave phase speed is computed over the central pacific (170°W to 120°W), and plotted against latitude in Figure 1. Also plotted are phase propagation observations provided by C. Hughes (2007, personal communication), zonally averaged over the same range. Speeds at latitudes 20°S and 20°N are well captured by the classic long Rossby wave solution. However departures are observed at both low latitudes and high latitudes. Observed speeds reach a maximum near $\pm 5°$. Poleward of 20° the Rossby wave solution diverges from the observations, reaching roughly a factor of two [Chelton and Schlax, 1996], and eastward propagation in the ACC region is also not captured. Figure 2 shows global maps of phase speed, “wavelikeness” and amplitude from Hughes’ dataset. Wavelikeness measures the precision of the distribution of phase speeds computed via a Radon transform at a given latitude, so one can already see from Figure 2 that low latitudes propagate mostly at coherent phase speeds while high latitudes exhibit a larger spread of propagation speeds, likely indicating a more turbulent flow.

A global map of the deformation radii used to calculate the theoretical long-wave phase speeds in Figure 1 is shown in Figure 3, and was obtained using the Forget [2008] atlas. The vertical
structure of the first baroclinic normal mode is plotted on the right for selected latitudes at 150°W in the Pacific Ocean, color-coded by the color of the x’s and using solid (dashed) lines in the southern (northern) hemisphere respectively. The stratification tends to be more surface intensified at lower latitudes, where $\Phi_1(z = 0)$ tends toward values near 4, and less surface intensified at high latitudes, where $\Phi_1(z = 0)$ is between 2 and 3. Note that the colormap saturates near the equator as deformation radii tend towards infinity.

### 2.2. Observations of oceanic currents and QGPV gradients

Figure 4 shows zonal averages of mean geostrophic zonal velocity and the meridional QGPV gradient $Q_y$ from the Forget [2008] atlas, with a black contour marking zero. Note that the QGPV gradient is nondimensionalized by the value of the planetary vorticity gradient at 30°, and that colors are saturated in the $Q_y$ plot. The important point to note is that $\nabla Q$ is clearly not well approximated by $\beta$. The salient features of the $Q_y$ plot include: (1) the zero crossing at 1 km depth in the ACC, just below the zonal jet which is responsible for significant baroclinically unstable growth and a steering level at depth, as reported in Smith and Marshall [2008], (2) the near surface zero crossings at low latitudes may contain baroclinic Charney instabilities, (3) the western boundary currents near 40°N (and the zero crossings below them), and (4) the convectively unstable regions in high latitudes where bottom water formation occurs.

### 2.3. Applicability of linear theory

We now consider the effects of including mean flow ($U$ and $\nabla Q$), estimated from the Forget [2008] atlas, by using the dispersion relationship (4) then setting $K = 0$ (i.e. the long-wave approximation). For each location we choose the vertical shear mode $\hat{\psi}_n$ whose real part projects the most onto the first neutral mode $\Phi_z(z)$ after its mean is subtracted and it is normalized.
The zonally averaged (from 170°W to 120°W) phase speeds are represented by the solid gray line in Figure 5. The observed central Pacific phase speeds from Figure 1 are also replotted for comparison. The long-wave limit predicts speeds which are too fast in low latitudes and typically (but not always) too slow in high latitudes. It is pleasing, however, to now observe eastward propagation in the ACC, a consequence of downstream advection by the mean current.

The assumed spatial scale of the waves also affects the predicted phase speeds. The same computation described above, but with deformation-scale waves ($K = K_1 \hat{x}$), gives the dashed gray line in Figure 5. Assuming the deformation scale as a lower limit for the wavelength of the observed waves, the solid and dashed lines in Figure 5 bracket the range of values one can obtain for the phase speed from linear theory. We address this range of possibilities more fully in the next section.

3. Fitting linear model phase speeds to observations

Traditionally, the long-wave approximation has been used when interpreting altimetric signals in terms of Rossby wave theory. The influence of horizontal scale on Rossby wave speed has largely been neglected, except for calculations assuming uniform wavelengths of 500 km and 200 km reported in Killworth and Blundell [2005]. Chelton et al. [2007] argue that the propagation of the observed SSH variability is due to eddies rather than Rossby waves, and remark that, equatorward of 25°, eddy speeds are slower than the zonal phase speeds of nondispersive baroclinic Rossby waves predicted by the long-wave theory. Here we show, however, that such a difference in speed can be accounted for by linear Rossby waves when their wavelengths are chosen appropriately.

Using equation (4) in its most general form, including bottom topography, Figure 6 shows both the best-fit phase speeds (left) and the wavelengths associated with those phase speeds.
(right) for a zonal average from 170°W to 120°W in the Pacific (top) and a global zonal average (bottom). We have assumed that the fitted waves have an east-west orientation \((\ell = 0)\). Setting \(k = \ell\) makes little difference in the fitted wavelength, which is consistent with the finding in Killworth and Blundell [2005] of a weak dependence of phase velocity on orientation. In the fitted wavelengths plots, the black x's correspond to individual latitudes, the solid gray curve is a smoother version of the black x's\(^6\), and the thin black line is the first deformation wavelength.

The fitted wavelengths typically lie between 600 km and 800 km in the tropics out to about 30°, with little or no dependence on the deformation wavelength. Note that the baroclinic Rhines scale (not shown) is roughly constant in the tropics, with a wavelength between 500km and 700km, and diverges to infinity when the turbulent velocity surpasses the longwave resting phase speed near ±20° (see below). There is a gap in fitted wavelength around ±40° where the linear theory fails to capture the observed phase speeds. At high latitudes, the best-fit is obtained assuming scales near the deformation scale. The inability to fit the phase speeds at higher latitudes is suggestive that the ‘wave’ signal is not linear in those regions. Clearly, though, the inclusion of wavelengths that result from a best-fit of theoretical to observed phase speeds results in a greatly improved prediction.

Figure 7 shows the importance of the planetary vorticity gradient \(\beta\) relative to the effect of mean flow \(U\) on the mean QGPV gradient \(\nabla Q\). Using the length scales computed by the best-fit algorithm, we plot the phase speeds that result from setting \(\beta = 0\) while keeping the observed \(U\) (thick dashed-dotted line), as well as the phase speeds that result from setting \(U = 0\) and \(\nabla Q = \beta \hat{y}\) (thin dashed line). [The solid gray line and black x’s are the same as those plotted in the upper left panel of Figure 6.] The planetary gradient \(\beta\) is crucial in the tropics, while in the subtropics \(U\) becomes increasingly important, particularly from 35°S to 20°S, where
the mean shear accounts for much of the factor-of-two phase speed error discussed in Chelton and Schlax [1996]. At high latitudes the Doppler shift caused by $U$ is crucial in capturing the downstream propagation in the ACC. Figure 7 also shows the effect of bottom topography on phase speed. Killworth and Blundell [2003] and Maharaj et al. [2007] showed that topography is only important in the presence of a mean flow. Here the best fit phase speeds with mean flow and a flat bottom (thin black line) are compared with the best fit speeds with topography (thick gray line). The fit is slightly better from $40^\circ$ to $50^\circ$ but the addition of topography is still not enough to completely fit the observations.

4. Wavelike and turbulent regimes in the ocean

A plausible interpretation of the results presented in Section 3 is that in low latitudes, baroclinic eddies give their energy to linear Rossby waves, whereas at high latitudes Rossby waves are less easily generated, and the SSH field remains dominated by eddies. This can be understood in terms of a matching, or otherwise, of turbulent and wave timescales, as discussed in the barotropic context by Rhines [1975] and Vallis and Maltrud [1993], and in a (first-mode) baroclinic context applied to the gas planets by Theiss [2004], Smith [2004] and Theiss [2006]. The central idea of the Rhines effect is that, as eddies grow in the inverse cascade, their timescale slows, and when this timescale matches the frequency of Rossby waves with the same spatial scale, turbulent energy may be converted into waves, and the cascade will slow tremendously. When this idea is applied to a putative interaction with baroclinic Rossby waves, there is the added complication that frequencies tend toward 0 at large scale (see Figure 8). In this case, only sufficiently weak eddies have timescales, at any wavelength, that intersect the Rossby wave dispersion curve.

For illustrative purposes, one can estimate the wavenumber at which the intersection occurs by assuming a turbulent dispersion relationship of the form $\omega_t = ku_t$, where $u_t$ is the turbulent
velocity scale (the square root of the appropriate eddy kinetic energy). Setting this equal to the absolute value of the approximate Rossby wave frequency \( \omega_R \simeq -k Q_y / (K^2 + K_1^2) \) assuming that \( Q_x \) is small and \( U \) is either small or constant in \( z \), we have (dividing by \( k \))

\[
u_t \sim \frac{Q_y}{K_1^2 + K^2}.
\] (7)

Solving for \( K \) gives the relationship \( K^2 = Q_y / u_t - K_1^2 \), for which there is a real solution only if \( Q_y / u_t > K_1^2 \). At fixed \( Q_y \) and \( K_1 \), the implication is that waves can be generated (and the cascade inhibited) only when the turbulent energy is sufficiently small. On the other hand, assuming a constant \( u_t \), and noting that \( Q_y \) (through its dependence on \( \beta \)) and \( K_1 \) (which is proportional to \( f \)) are dependent on latitude, the relationship (7) implies the existence of a critical latitude, polewards of which no intersection is possible.

Let us now see what the data suggests about a relationship like (7). We replace the approximate Rossby wave dispersion relation with the frequencies from (4), using the fitted Rossby wave scales described in the previous section. The idea is illustrated in Figure 8, which shows [Fig. 8] zonally averaged Rossby wave frequency curves \( \omega_R(k) \), plotted against zonal wavelength, at three latitudes in the tropical Pacific Ocean. Two hypothetical eddy frequency curves \( \omega_t = k u_t \) (dashed lines) are added for comparison, with \( u_t = 10 \text{ cm s}^{-1} \) and \( u_t = 5 \text{ cm s}^{-1} \). At 10°S the eddy frequency curves intersect the Rossby wave frequencies at relatively small wavelengths, indicating that observed tropical SSH length scales are certainly in the wave region. On the other hand, at 30°S even the 5 cm s\(^{-1}\) curve fails to intersect \( \omega_R(k) \). We thus expect little wavelike activity outside the tropics.

We can improve the frequency comparison test further by using observations of surface drifter speeds to obtain estimates of \( u_t \). A global map of the root mean square (rms) of the surface
drifter data

\[ u_{\text{rms}}(0) = \sqrt{|u'_{\text{drifter}}(z = 0)|^2} \]

(courtesy of Nikolai Maximenko) is shown Figure 9, with its zonal average over 170°W to 120°W (the region within the rectangle) plotted on the right. The zonal average is strongly peaked at the equator, and more constant at extra-tropical latitudes. However, this may not be indicative of the distribution of total eddy kinetic energy, since the surface velocity gives no information about the vertical structure of eddying motion. Additional assumptions are necessary to extract the relevant eddy velocity scale.

Wunsch [1997] showed that, away from the equator, eddy velocities are primarily first-baroclinic, with a smaller projection onto the barotropic mode, while nearer the equator, motions tend to have a more complex vertical structure, projecting onto many higher modes, approaching equipartition. Expanding \( u_{\text{rms}}(z) \) in the neutral modes (6), we have

\[ u_{\text{rms}}(z) = \sum_{m=0}^{N_x} \Phi_m(z) u_m. \]

Following Wunsch [1997], we extract the vertical structure at each location by assuming that the \textit{rms velocity projects entirely onto the first baroclinic mode}, which gives \( u_1 = u_{\text{rms}}(0)/\Phi_1(0) \).

Since we are considering first baroclinic Rossby waves, the projection \( u_1 \) is the relevant eddy velocity scale, which is also the root vertical-mean square velocity (if the flow is entirely first baroclinic), thus

\[ u_1 = \left[ \frac{1}{H} \int_{-H}^{0} u_{\text{rms}}(z)^2 \, dz \right]^{1/2} = u_{\text{rms}}(0)/\Phi_1(0) \]

where we have used the orthonormality of the neutral modes.\(^7\) The scaling by the first baroclinic mode has the effect of reducing the estimated turbulent velocity scale in regions of strongly surface intensified stratification, such as near the equator. In these regions, the first baroclinic mode itself is quite surface intensified, so \( \Phi_1(0) \) can be considerably larger than one (see the
modal structure in Figure 3). Physically, if the first neutral mode, onto which all the motion is assumed to project, is very surface intensified, then eddy velocities are weak at depth, so the total turbulent velocity estimate is diminished.

Figure 10 shows the eddy velocity scale $u_t$ and zonal Rossby phase speed $c_R$ zonally averaged over $170^\circ W$ to $120^\circ W$ and plotted against latitude. These are essentially the left hand and equivalent right hand sides of Eq. (7). Our Figure 10 is similar to Fig. 3 of Theiss [2006] for Jupiter, except that here our dispersion relation is computed from the full vertical structure of the mean flow, rather than just the first baroclinic component (because of the dominance of the first baroclinic mode, however, the first baroclinic calculation is rather similar — not shown). Note that $u_t$ is nearly constant with latitude, varying between and 5 and 10 cm s$^{-1}$ — the strong equatorial values have been reduced, through projection onto the surface-intensified first baroclinic mode, as explained above (if one assumed equipartition, the velocity estimate in the equatorial region would be reduced even further). In contrast, the (Doppler-shifted) Rossby wave speed varies markedly, exceeding 20 cm s$^{-1}$ in the tropics and falling toward zero at higher latitudes (and even becoming prograde in the ACC). The cross-over between the two curves occurs at a latitude of roughly $\pm 25^\circ$. Note that since we have assumed that the turbulent velocity scale $u_t$ is entirely in the first baroclinic mode, the crossover latitudes should be considered as lower bounds.

The lower plot in Figure 10 shows the ratio of linear phase speeds $c_R$ to the eddy velocity scale $u_t$, with dashed lines denoting $c_R/u_t = 2$ and $1/2$. Theiss [2006] shows that stormy regions on Jupiter are highly correlated with regions where this ratio is less than one. Notably, $\pm 25^\circ$ is also the crossover latitude between linear wavelike behavior and nonlinear eddies found by Chelton et al. [2007]. Outside this latitude band, first-baroclinic Rossby wave timescales cannot match
the turbulent timescales implied by $u_t$. Note that this would not preclude the formation of the mid-latitude zonal jets observed by Maximenko et al. [2005] and Richards et al. [2006]: since barotropic Rossby waves are possible, turbulent energy can still accumulate around the dumbbell of Vallis and Maltrud [1993].

Finally we return to a consideration of the spatial scales obtained by fitting linear Rossby wave theory to observed phase speeds, as in Figure 6. A global zonal average of the fitted wavelengths is plotted against latitude in Figure 11. Also plotted are both observed (black o’s) and simulated (black x’s) eddy wavelengths in the North Atlantic from Eden [2007], as well as globally observed wavelengths (small circles with line) from Chelton et al. [2007]. The deformation wavelength (thin black line) is also plotted for reference. Note that the baroclinic Rhines wavelength (not shown) is also of the order of 600-700km in the low latitudes, but diverges to infinity near $\pm 25^\circ$ where $c_R = u_t$ in Figure 10 so there is no baroclinic Rhines scale outside of this latitude band.

At low latitudes all of the scales are in close agreement, while the fitted wavelength diverges from the observed eddy scales at latitudes poleward of about $\pm 40^\circ$. This is also near the latitude where Eden’s scales transition from a flatter Rhines scaling to a steeper deformation scaling. In the Southern Ocean there is a transition from westward propagation to eastward propagation upon entering the ACC region. Finally we note that, in contrast to Eden [2007], Chelton’s data do not exhibit a clear transitional latitude between Rhines scaling and deformation scaling. The reasons for this remain unclear.

5. Conclusions

We have revisited the interpretation of altimetric phase speed signals in terms of linear Rossby wave theory. Given observations of the interior $u$ and $\nabla Q$ fields [courtesy of Forget, 2008], and assuming quasi-geostrophic theory, we adjusted the lateral scale of linear waves to best fit
altimetric observations of westward phase propagation. We find that the implied scales have a
well-defined meridional structure. In low latitudes the waves have a scale of 600 km or so, broadly
consistent with an appropriately defined Rhines scale. In high latitudes it is more difficult to fit
linear theory to the observations, but our attempts to do so imply a scale that is much smaller than
in the tropics, closer to the local Rossby deformation scale. There is a rather abrupt transition
from low-latitude to high-latitude scaling at $\pm 30^\circ$. These results are broadly consistent with
observed and modeled eddy scales, as reported in Eden [2007].

We put forward an interpretation of the reported results in terms of the interaction between
turbulence and waves. Over vast regions of the ocean, at scales on or close to the Rossby
deformation scale, baroclinic instability converts available potential energy to kinetic energy of
turbulent geostrophic motion. Non-linear interactions result in an upscale energy transfer. At low
latitudes, where we observe that $u_t < |c_R|$, turbulent energy cascades upscale from below readily
excites Rossby waves. At higher latitudes, where $u_t > |c_R|$, turbulence cannot readily excite
waves because of the weak overlap in timescales between turbulence and waves. Making use of
surface drifter observations, we estimate that the latitude at which waves give way to turbulence
coincides with that at which $u_t \sim |c_R|$, and is found to be $\pm 30^\circ$ or so, roughly consistent with
the transition from waves to non-linear eddies recently highlighted by Chelton et al. [2007].

Appendix: Mean State Calculation from the Forget Atlas and Discretization of Linear
Problem

The Forget [2008] ocean atlas contains up to 50 layers (of thicknesses $\Delta_j$) of potential temper-
ature and salinity data at each (lat, lon) coordinate. We first compute annually averaged global
potential temperature and salinity fields, and from these compute a neutral density field $\tilde{\rho}$ using
locally referenced pressure. Thermal wind balance is then used to compute the mean velocity
field $U$, assuming a level of no motion at the bottom of the ocean [see the appendix of Smith, 2007, for details]. We define the top 5 layers, which are each 10 m thick, as a mixed layer of depth $h \equiv 50$ m. The mean buoyancy gradients $\nabla B = -(g/\rho_0) \nabla \theta$ at the surface are averaged over the defined mixed layer, and then related to vertical shears via thermal wind

$$U_z(z_0) = -\frac{1}{f h} \int_{-h}^{0} B_y dz, \quad V_z(z_0) = \frac{1}{f h} \int_{-h}^{0} B_x dz.$$  

The surface velocities themselves are obtained by averaging the velocities from the ocean atlas over $h$, viz.

$$U(z_0) = \frac{1}{h} \int_{-h}^{0} U dz, \quad V(z_0) = \frac{1}{h} \int_{-h}^{0} V dz.$$  

(A1)

The linear problem is discretized, at each lateral location, onto the $N_z$ discrete depths $z_j$ of the data computed from the Forget atlas. The discrete surface buoyancy is given by

$$\hat{b}_m(z_0) = f \frac{\hat{\psi}_m(z_0) - \hat{\psi}_m(z_1)}{\Delta_0}.$$  

and the discrete PV is

$$\hat{q}_m(z_j) = \frac{f^2}{\Delta_j} \left[ \frac{\hat{\psi}_m(z_{j-1}) - \hat{\psi}_m(z_j)}{B(z_{j-1}) - B(z_j)} - \frac{\hat{\psi}_m(z_j) - \hat{\psi}_m(z_{j+1})}{B(z_j) - B(z_{j+1})} \right] - K^2 \hat{\psi}_m(z_j), \quad j = 1..N_z - 1.$$  

The mean QGPV gradients $Q_x(z_j)$ and $Q_y(z_j)$ are given by (2), using the same vertical discretization, and simple horizontal finite differences to compute $x$ and $y$ derivatives. At the bottom, $\hat{q}_m(z_{N_z}) = \hat{\psi}_m(z_{N_z}) = 0$. The discrete version of (4) is then solved as a single matrix eigenvalue problem, using Matlab.

Following Smith [2007], bottom topography is added using the Smith and Sandwell [1997] global seafloor topography dataset. At each latitude, longitude location in the calculation we linearly regress a best fit plane of the form $\eta(x, y) = \eta_0 + \alpha^x x + \alpha^y y$ using the surrounding $2^\circ \times 2^\circ$ section of topography. The slopes $\alpha^x$ and $\alpha^y$ are then added to the bottom (layer $N$).
QGPV gradient as

\[ \nabla Q^{\text{topo}} = \frac{f}{\Delta N} (\alpha x \hat{x} + \alpha y \hat{y}) \].

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Notes

1. The inverse cascade observed by Scott and Wang [2005] presented a conundrum since up to 70% of the variability at the ocean surface is contained in the first baroclinic mode [Wunsch, 1997] and it was thought that first baroclinic energy should cascade towards the Rossby radius. However Scott and Arbic [2007] showed in simulations of two-layer baroclinic turbulence, that while the total energy in the first baroclinic mode cascades towards the Rossby radius, the kinetic energy moves upscale.

2. Horizontal shears of the mean state contribute little to \( \nabla Q \) [Smith, 2007], but we retain them in our calculations for completeness.

3. The observed propagation speeds were calculated by Hughes from SSH observations in the following way. First, thin longitude (5 degrees) and tall time (11.5 years) strips are bandpassed filtered in time from 5 to 57 weeks, then zonally averaged (at each time) and the annual and semianual cycles are removed. A Radon transform was then performed by shifting each longitude such that signals traveling at a speed \( c \) line up horizontally, summing over longitude and taking the standard deviation in time. A “wavelikeness” parameter is also computed as the peak value of the Radon transform divided by its mean. Based on advice from Hughes we have filtered out observations with wavelikeness less than 1.5. Figure 2 shows global maps of the observed phase speed, wavelikeness (with black contour at 1.5), and a measure of wave amplitude.

4. The region above the dashed line indicates the layers from \( z = 0 \) to \( z = -h \) that were averaged over in order to compute the upper PV-sheet \( f^2 U_z(z_0)/N^2 \), as described in the appendix.

5. Specifically, we choose \( \hat{\psi}_n \) such that

\[ \max_n \int \Phi_1 \left( \hat{\psi}_n - \bar{\psi}_n \right) dz / \int \left( \hat{\psi}_n - \bar{\psi}_n \right)^2 dz \].

6. The fitted wavelengths are smoothed across latitudes using a 1-1-1 smoother defined by:

\[ \lambda'_i = (\lambda_{i-1} + \lambda_i + \lambda_{i+1})/3 \],

where \( \lambda_i \) is the wavelength at latitude \( i \) and \( \lambda'_i \) is the smoothed value.
7. Suppose, instead of assuming that all the energy was in the first baroclinic mode, we imagined that \( U(z) \) projected equally onto the barotropic and first baroclinic mode. Then

\[
\begin{align*}
\bar{u}_t &= \left[ \frac{1}{H} \int u_{\text{rms}}(z)^2 \, dz \right]^{1/2} \\
&= \frac{u_{\text{rms}}(0)}{1 + \Phi_1(0)} \left[ \frac{1}{H} \int (1 + \Phi_1(z))^2 \, dz \right]^{1/2} \\
&= \sqrt{2}u_{\text{rms}}(0) \frac{1 + \Phi_1(0)}{1 + \Phi_1(0)},
\end{align*}
\]

since the modes are orthonormal. In the world ocean \( 2 \leq \Phi_1(0) \leq 4 \), so the ratio of this projected value to one which is entirely first baroclinic, as assumed in the text, is \( 0.94 \leq \sqrt{2}\Phi_1(0)/[1 + \Phi_1(0)] \leq 1.13 \). An assumption of equipartition among \( N_z \) vertical modes unambiguously reduces \( \bar{u}_t \), roughly by a factor of roughly \( \sqrt{N_z} \).

8. Chelton provides eddy diameters, and here these are multiplied by \( \pi \) to give wavelengths.

References


Figure 1. Westward phase speed estimated from Hughes’ data averaged from 170°W to 120°W (black x’s) plotted against the standard linear, first baroclinic, long Rossby wave phase speed (solid line), computed from the Forget [2008] atlas.
Figure 2. Hughes’ analysis of surface altimetric data. Phase speed (top), with a white contour at 0, to differentiate westward and eastward propagating regions, “wavelikeness” (middle — see text for details), with contour at 1.5 to differentiate regions that are wavelike and not wavelike, and a measure of amplitude (bottom).
Figure 3. Map of first internal deformation radius (left), vertical structure of the first baroclinic mode (right), $\Phi_1(z)$, at the positions marked with colored x’s (at latitudes 60.5°S, 45.5°S, 30.5°S, 15.5°S, 0.5°S, 14.5°N, 29.5°N, and 44.5°N, and longitude 150°W). The lines are color-coded with dashed lines indicating the northern hemisphere and solid lines the southern hemisphere.
Figure 4. Mean zonal velocity $U$ (top), zonally averaged from $170^\circ W$ to $120^\circ W$ in the Pacific, and meridional QGPV gradient (bottom) zonally averaged over the same region. The PV gradient is normalized by the value of the planetary vorticity gradient, $\beta$, at 30 degrees. Note that the zero contour is indicated by black contours and that the color axis is saturated. The regions above the horizontal dashed line indicate the PV-sheet layer.
Figure 5. Hughes’ phase speed observations (black x’s) compared to linear theory in the presence of a mean current: long-waves (gray solid line) and deformation scale waves (gray dashed line).
Figure 6. Top left: Phase speeds according to linear theory (solid gray line) adjusted to give the best match to Hughes’ data (black x’s). The fit is done for a zonal average over 170°W to 120°W in the Pacific. Top right: Fitted wavelengths at each latitude (black x’s, gray line is a smoothed version) along with the deformation scale (thin solid line). Bottom panels: As in the top panels but zonally averaged across all oceans.
Figure 7. Comparison of the effects of $\beta$, mean currents, and topography. The x’s and thick gray solid line are identical to those in the top left panel of Figure 6 (zonal average over the Pacific region 170°W to 120°W). The thin black line shows the best fit phase speed with nonzero $U$ and $\beta$ but no topography, the thin dashed line corresponds to nonzero $\beta$, $U = 0$ and no topography, and the thick dash-dotted line corresponds to nonzero $U$, $\beta = 0$, and no topography. In all cases the best fit horizontal scale of Figure 6 is used.
Figure 8. Dispersion relations for fitted phase speeds as a function of zonal wavelength (with meridional wavenumber $\ell = 0$) for latitudes in the South Pacific ($10^\circ S$, $20^\circ S$ and $30^\circ S$), compared with $\omega_t = ku_t$ with two values of $u_t$: 5 and 10 cm s$^{-1}$ (dashed lines).
Figure 9. Root mean square eddying surface velocities (left) from N. Maximenko’s drifter data, and zonal average thereof (right).
Figure 10. Top: Doppler shifted long-wave phase speed (thin black line), versus the root mean square of the eddy velocity $u_t$ (thick gray line) from Maximenko’s drifter data. It has been assumed that the eddy velocity is entirely in the first baroclinic mode. Bottom: The ratio $c_R/u_t$ with dashed curves at ratios $1/2$ and $2$. 


Figure 11. Comparison of fitted wavelengths over the global ocean (gray curve, taken from the bottom-right of Figure 6) against Eden’s observed (black o’s) and simulated (black x’s) North Atlantic wavelengths ($2\pi$ times the values in his Figure 7a), Chelton’s globally observed wavelengths ($\pi$ times eddy diameter, small black circles with solid line), and the deformation wavelength (thin black line).